

## Mathematics Benchmarks - Grade 5

The Newton Public Schools has fully transitioned to the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards*. The new standards, developed by education officials from 48 states, address the ongoing concerns at the national level about the focus and coherence of mathematics education, and about the depth of understanding of mathematics among students in the United States.

The focus of the new frameworks requires that we change the content of our curriculum by decreasing the number of topics to be taught each year, and by deliberately building concepts step-by-step from grade to grade. Increasing the depth of student understanding of mathematics requires changing some of our traditional instructional strategies.

With the transition, we adjusted mathematical content and our mathematical practices simultaneously. We continue to use *Everyday Mathematics* as our core text, eliminating topics that are no longer content expectations in fifth grade, adding materials that deepen the instruction on topics defined as critical areas for fifth grade, and adding materials that address topics not included in fifth grade *Everyday Math*.

In addition to providing new content, our new materials define a management structure to help teachers engage all children in the mathematics they are learning: in reasoning, problem solving, communicating their mathematical thinking and critiquing the reasoning of others - all mathematical practices identified in the Common Core State Standards.

It is our firm belief that the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards* - built on a foundation of mathematical practices and instructional strategies that deliberately and specifically promote student engagement with mathematics - will further the system-wide core value of Respect for Human Differences. With these frameworks as a structure, teachers will have additional tools for developing students' skills in:

- carefully considering the mathematical thinking of peers of diverse mathematical backgrounds and interests.
- collaborating in problem-solving with peers of a different race, ethnicity, gender, and cultural tradition.
- incorporating the points of view of others into their analysis and understanding.
- identifying ways in which our present knowledge results from the work of women and men from different cultures over many centuries.
- recognizing that their own success in mathematics is not diminished when others succeed.

The complete text of the new frameworks can be downloaded from the Massachusetts Department of Elementary and Secondary Education website: [www.doe.mass.edu/frameworks/current.html](http://www.doe.mass.edu/frameworks/current.html). Further information about the Common Core State Standards Initiative can be found at [www.corestandards.org](http://www.corestandards.org)

## Mathematical Practices - Grade 5

Mathematical practices students will use in fifth grade include:

1. Making sense of problems and persevering in solving them.  
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reasoning abstractly and quantitatively.  
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Constructing viable arguments and critiquing the reasoning of others.  
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Modeling with mathematics.  
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Using appropriate tools strategically.  
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attending to precision.  
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other.
7. Looking for and making use of structure.  
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property.
8. Looking for and expressing regularity in repeated reasoning.  
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Mathematical Content - Grade 5

### Critical Areas:

Instructional time in fifth grade will concentrate on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

- (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations

with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

- (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
- (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

## Content Standards - Grade 5

### **Operations and Algebraic Thinking**

### **5.OA**

#### **Write and interpret numerical expressions.**

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "Add 8 and 7, then multiply by 2" as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.*

#### **Analyze patterns and relationships.**

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given*

the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

## Number and Operations in Base Ten

5.NBT

### Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $\frac{1}{10}$  of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
  - a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,  
 $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$ .
  - b. Compare two decimals to thousandths based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

### Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

## Number and Operations—Fractions

5.NF

### Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ .*  
(In general,  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ .)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$ , by observing that  $\frac{3}{7} < \frac{1}{2}$ .*

### Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ( $\frac{a}{b} = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret  $\frac{3}{4}$  as the result of dividing 3 by 4, noting*

- that  $\frac{3}{4}$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $\frac{3}{4}$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
- 4.\* Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
    - a. Interpret the product  $(\frac{a}{b}) \times q$  as  $a$  parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(\frac{2}{3}) \times 4 = \frac{8}{3}$ , and create a story context for this equation. Do the same with  $(\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}$ . (In general,  $(\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}$ .)*
    - b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
  - 5.\* Interpret multiplication as scaling (resizing), by:
    - a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
    - b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$  to the effect of multiplying  $\frac{a}{b}$  by 1.
  6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
  7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<sup>1</sup>
    - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for  $(\frac{1}{3}) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(\frac{1}{3}) \div 4 = \frac{1}{12}$  because  $(\frac{1}{12}) \times 4 = \frac{1}{3}$ .*
    - b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for  $4 \div (\frac{1}{5})$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (\frac{1}{5}) = 20$  because  $20 \times (\frac{1}{5}) = 4$ .*
    - c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share  $\frac{1}{2}$  lb of chocolate equally? How many  $\frac{1}{3}$ -cup servings are in 2 cups of raisins?*

## The Number System

5.NS

### Gain familiarity with concepts of positive and negative integers.

MA.1. Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.

<sup>1</sup> Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

\* Standards 4 and 5 are not assessable on the 2013 MCAS.

**Convert like measurement units within a given measurement system.**

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

**Represent and interpret data.**

2. Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

**Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
  - a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
  - b. A solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
  - a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
  - b. Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
  - c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

**Graph points on the coordinate plane to solve real-world and mathematical problems.**

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,  $x$ -axis and  $x$ -coordinate,  $y$ -axis and  $y$ -coordinate).
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

**Classify two-dimensional figures into categories based on their properties.**

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.