

## Mathematics Benchmarks - Grade 4

The Newton Public Schools has fully transitioned to the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards*. The new standards, developed by education officials from 48 states, address the ongoing concerns at the national level about the focus and coherence of mathematics education, and about the depth of understanding of mathematics among students in the United States.

The focus of the new frameworks requires that we change the content of our curriculum by decreasing the number of topics to be taught each year, and by deliberately building concepts step-by-step from grade to grade. Increasing the depth of student understanding of mathematics requires changing some of our traditional instructional strategies.

With the transition, we adjusted mathematical content and our mathematical practices simultaneously. We continue to use *Everyday Mathematics* as our core text, eliminating topics that are no longer content expectations in fourth grade, adding materials that deepen the instruction on topics defined as critical areas for fourth grade, and adding materials that address topics not included in fourth grade *Everyday Math*.

In addition to providing new content, our new materials define a management structure to help teachers engage all children in the mathematics they are learning: in reasoning, problem solving, communicating their mathematical thinking and critiquing the reasoning of others - all mathematical practices identified in the Common Core State Standards.

It is our firm belief that the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards* - built on a foundation of mathematical practices and instructional strategies that deliberately and specifically promote student engagement with mathematics - will further the system-wide core value of Respect for Human Differences. With these frameworks as a structure, teachers will have additional tools for developing students' skills in:

- recognizing that our present mathematical knowledge results from the work of women and men from different cultures over many centuries
- carefully considering the mathematical thinking of peers of diverse mathematical backgrounds and interests.
- collaborating in problem-solving with peers of a different race, ethnicity, gender, and cultural tradition.
- recognizing that one's success in mathematics is not diminished when others succeed.

The complete text of the new frameworks can be downloaded from the Massachusetts Department of Elementary and Secondary Education website: [www.doe.mass.edu/frameworks/current.html](http://www.doe.mass.edu/frameworks/current.html). Further information about the Common Core State Standards Initiative can be found at [www.corestandards.org](http://www.corestandards.org)

## Mathematical Practices - Grade 4

Mathematical practices students will use in fourth grade include:

1. Making sense of problems and persevering in solving them.  
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reasoning abstractly and quantitatively.  
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Constructing viable arguments and critiquing the reasoning of others.  
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Modeling with mathematics.  
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Using appropriate tools strategically.  
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attending to precision.  
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other.
7. Looking for and making use of structure.  
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property.
8. Looking for and expressing regularity in repeated reasoning.  
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Mathematical Content - Grade 4

### Critical Areas:

Instructional time in fourth grade will concentrate on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

- (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of

operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

- (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,  $\frac{15}{9} = \frac{5}{3}$ ), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

## Content Standards - Grade 4

### Operations and Algebraic Thinking

4.OA

#### Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
3. Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

#### Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

### Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

## Number and Operations in Base Ten<sup>1</sup>

4.NBT

### Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that  $700 \div 70 = 10$  by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

### Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  
MA.5a. Know multiplication facts and related division facts through  $12 \times 12$ .
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

## Number and Operations—Fractions<sup>2</sup>

4.NF

### Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .
  - a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

<sup>1</sup> Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

<sup>2</sup> Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:  $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ ;  $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$ ;  $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$ .*
  - c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
  - d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. *Not a standard until 2013-2014.*

**Understand decimal notation for fractions, and compare decimal fractions.**

- 5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.<sup>3</sup> *For example, express  $\frac{3}{10}$  as  $\frac{30}{100}$ , and add  $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$ .*
- 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as  $\frac{62}{100}$ ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
- 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model.

**Measurement and Data**

**4.MD**

**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

- 1. Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
- 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
- 3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

**Represent and interpret data.**

- 4. Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
- 5, 6, 7. *Not standards until 2013-2014.*

<sup>3</sup> Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

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**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.