

Mathematics Benchmarks - Grade 3

The Newton Public Schools has fully transitioned to the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards*. The new standards, developed by education officials from 48 states, address the ongoing concerns at the national level about the focus and coherence of mathematics education, and about the depth of understanding of mathematics among students in the United States.

The focus of the new frameworks requires that we change the content of our curriculum by decreasing the number of topics to be taught each year, and by deliberately building concepts step-by-step from grade to grade. Increasing the depth of student understanding of mathematics requires changing some of our traditional instructional strategies.

With the transition, we adjusted mathematical content and our mathematical practices simultaneously. We continue to use *Everyday Mathematics* as our core text, eliminating topics that are no longer content expectations in third grade, adding materials that deepen the instruction on topics defined as critical areas for third grade, and adding materials that address topics not included in third grade *Everyday Math*.

In addition to providing new content, our new materials define a management structure to help teachers engage all children in the mathematics they are learning: in reasoning, problem solving, communicating their mathematical thinking and critiquing the reasoning of others - all mathematical practices identified in the Common Core State Standards.

It is our firm belief that the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards* - built on a foundation of mathematical practices and instructional strategies that deliberately and specifically promote student engagement with mathematics - will further the system-wide core value of Respect for Human Differences. With these frameworks as a structure, teachers will have additional tools for developing students' skills in:

- carefully considering the mathematical thinking of peers of diverse mathematical backgrounds and interests.
- collaborating in problem-solving with peers of a different race, ethnicity, gender, and cultural tradition.
- incorporating the points of view of others into their analysis and understanding.
- identifying ways in which our present knowledge results from the work of women and men from different cultures over many centuries.
- recognizing that their own success in mathematics is not diminished when others succeed.

The complete text of the new frameworks can be downloaded from the Massachusetts Department of Elementary and Secondary Education website:
www.doe.mass.edu/frameworks/current.html.

Mathematical Practices - Grade 3

Mathematical practices students will use in third grade include:

1. Making sense of problems and persevering in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reasoning abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Constructing viable arguments and critiquing the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Modeling with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Using appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attending to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other.
7. Looking for and making use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property.
8. Looking for and expressing regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematical Content - Grade 3

Critical Areas:

Instructional time in third grade will concentrate on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

- (1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

- (2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
- (3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
- (4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Content Standards - Grade 3

Operations and Algebraic Thinking

3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*
2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.*

Understand properties of multiplication and the relationship between multiplication and division.

- Apply properties of operations as strategies to multiply and divide.¹ *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$ then $15 \times 2 = 30$, or by $5 \times 2 = 10$ then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*
- Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.²
- Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.³

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

¹ Students need not use formal terms for these properties.

² This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

³ A range of algorithms may be used.

Develop understanding of fractions as numbers.

1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
 - a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.
 - b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
 - a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
 - b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
 - c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*
 - d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data**3.MD****Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).⁵ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

⁴ Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

⁵ Excludes compound units such as cm^3 and finding the geometric volume of a container.

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
 - a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
 - b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
 - a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
 - b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
 - c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
 - d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal areas and describe the area of each part as $\frac{1}{4}$ of the area of the shape.*

TABLE 2. Common multiplication and division situations.⁶

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$ and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,⁷ Area⁸	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

⁶ The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

⁷ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁸ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.