Mathematics Benchmarks - Grade 2

The Newton Public Schools has fully transitioned to the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards*. The new standards, developed by education officials from 48 states, address the ongoing concerns at the national level about the focus and coherence of mathematics education, and about the depth of understanding of mathematics among students in the United States.

The focus of the new frameworks requires that we change the content of our curriculum by decreasing the number of topics to be taught each year, and by deliberately building concepts step-by-step from grade to grade. Increasing the depth of student understanding of mathematics requires changing some of our traditional instructional strategies.

With the transition, we adjusted mathematical content and our mathematical practices simultaneously. We continue to use *Everyday Mathematics* as our core text, eliminating topics that are no longer content expectations in second grade, adding materials that deepen the instruction on topics defined as critical areas for second grade, and adding materials that address topics not included in second grade *Everyday Math*.

In addition to providing new content, our new materials define a management structure to help teachers engage all children in the mathematics they are learning: in reasoning, problem solving, communicating their mathematical thinking and critiquing the reasoning of others - all mathematical practices identified in the Common Core State Standards.

It is our firm belief that the 2011 *Massachusetts Mathematics Curriculum Frameworks, incorporating the Common Core State Standards* - built on a foundation of mathematical practices and instructional strategies that deliberately and specifically promote student engagement with mathematics - will further the system-wide core value of Respect for Human Differences. With these frameworks as a structure, teachers will have additional tools for developing students' skills in:

- valuing and incorporating different points of view.
- carefully considering the mathematical thinking of peers of diverse mathematical backgrounds and interests.
- collaborating in problem-solving with peers of a different race, ethnicity, gender, and cultural tradition.

The complete text of the new frameworks can be downloaded from the Massachusetts Department of Elementary and Secondary Education website: www.doe.mass.edu/frameworks/current.html. Further information about the Common Core State Standards Initiative can be found at www.corestandards.org

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Mathematical Practices - Grade 2

Mathematical practices students will use in second grade include:

1. Making sense of problems and persevering in solving them.
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reasoning abstractly and quantitatively.
   Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Constructing viable arguments and critiquing the reasoning of others.
   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They justify their conclusions, communicate them to others, and respond to the arguments of others. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Modeling with mathematics.
   Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Using appropriate tools strategically.
   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator. Proficient students are sufficiently familiar with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful.

6. Attending to precision.
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. In the elementary grades, students give carefully formulated explanations to each other.

7. Looking for and making use of structure.
   Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.

8. Looking for and expressing regularity in repeated reasoning.
   Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematical Content - Grade 2

Critical Areas:

Instructional time in second grade will concentrate on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the

properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Content Standards - Grade 2

Operations and Algebraic Thinking   2.OA

Represent and solve problems involving addition and subtraction.

1.   Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2.   Fluently add and subtract within 20 using mental strategies.[2] By end of grade 2, know from memory all sums of two one-digit numbers.
      MA.2.a. By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3.   Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

4.   Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

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[1] See Glossary, Table 1.
[2] See standard 1.OA.6 for a list of mental strategies.

Number and Operations in Base Ten   2.NBT

Understand place value.

1.   Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
    a. 100 can be thought of as a bundle of ten tens—called a "hundred."
    b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.   Count within 1000; skip-count by 5s, 10s, and 100s.
3.   Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4.   Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5.   Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6.   Add up to four two-digit numbers using strategies based on place value and properties of operations.
7.   Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8.   Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9.   Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

Measurement and Data   2.MD

Measure and estimate lengths in standard units.

1.   Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2.   Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3.   Estimate lengths using units of inches, feet, centimeters, and meters.
4.   Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5.   Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

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[3] Explanations may be supported by drawings or objects.

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
   MA.7.a. Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month, and a year; and weeks in a month and a year.

8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems,[4] using information presented in a bar graph.

Geometry   2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

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[4] See Glossary, Table 1.
[5] Sizes are compared directly or visually, not compared by measuring.

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 − 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 − ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? − 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[7]
Put Together/ Take Apart[8]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, \ 5 − 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[9]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, \ 5 − 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, \ 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 − 3 = ?, \ ? + 3 = 5$

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[6] Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32–33).

[7] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[8] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[9] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.