Model Advanced Course: Model Precalculus Overview [PC]

Number and Quantity

The Complex Number System

- **A.** Perform arithmetic operations with complex numbers.
- **B.** Represent complex numbers and their operations on the complex plane.
- **C.** Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities

- **A.** Represent and model with vector quantities.
- **B.** Perform operations on vectors.
- **C.** Perform operations on matrices and use matrices in applications.

Algebra

Arithmetic with Polynomials and Rational Expressions

- **C.** Use polynomial identities to solve problems
- **D.** Rewrite rational expressions.

Reasoning with Equations and Inequalities

- **C.** Solve systems of equations.

Functions

Interpreting Functions

- **C.** Analyze functions using different representations.

Building Functions

- **A.** Build a function that models a relationship between two quantities.
- **B.** Build new functions from existing functions.

Trigonometric Functions

- **A.** Extend the domain of trigonometric functions using the unit circle.
- **B.** Model periodic phenomena with trigonometric functions.
- **C.** Prove and apply trigonometric identities.

Geometry

Similarity, Right Triangles, and Trigonometry

- **D.** Apply trigonometry to general triangles.

Circles

- **A.** Understand and apply theorems about circles.

Expressing Geometric Properties with Equations

- **A.** Translate between the geometric description and the equation for a conic section.

Geometric Measurement and Dimension

- **A.** Explain volume formulas and use them to solve problems.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Model Advanced Course: Model Precalculus Content Standards [PC]

Number and Quantity

The Complex Number System PC.N-CN

A. Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

B. Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

For example, $(-1+\sqrt{3}i)^3=8$ because $(-1+\sqrt{3}i)$ has modulus 2 and argument 120°.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

C. Use complex numbers in polynomial identities and equations.

8. (+) Extend polynomial identities to the complex numbers.

For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities PC.N-VM

A. Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

B. Perform operations on vectors.

4. (+) Add and subtract vectors.

- **a.** (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **b.** (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **c.** (+) Understand vector subtraction $\boldsymbol{v} – \boldsymbol{w}$ as $\boldsymbol{v} + (–\boldsymbol{w})$, where $–\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.

- **a.** (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
- **b.** (+) Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

C. Perform operations on matrices and use matrices in applications.